

# Statistics

## Spring 2023

### Lecture 51



Feb 19-8:47 AM

Testing population standard deviation:

$$\begin{array}{l}
 H_0: \sigma = \sigma_0 \\
 H_1: \sigma \neq \sigma_0 \\
 \text{TTT}
 \end{array}
 \left.
 \begin{array}{l}
 H_0: \sigma \leq \sigma_0 \\
 H_1: \sigma > \sigma_0 \\
 \text{RTT}
 \end{array}
 \right\}
 \begin{array}{l}
 H_0: \sigma \geq \sigma_0 \\
 H_1: \sigma < \sigma_0 \\
 \text{LTT}
 \end{array}$$

P-value Method

$$\text{CTS } \chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2}$$

use  $\chi^2 \text{cdf}$  with  $df = n-1$

to find the p-value.

Drawing, shading, labeling  
Required

use testing chart to learn about  $H_0$  &  $H_1$ .

Draw Final Conclusion about the claim.

Reject the claim OR FTR the claim

May 16-7:03 AM

Given:  $n=12$ ,  $S=15$ ,  $H_0: \sigma \leq 10$ , claim is  $H_1$

Test the claim.  $\Rightarrow$  NO  $\alpha \Rightarrow$  use .05

$H_0: \sigma \leq 10$

$H_1: \sigma > 10$  claim, RTT

CTS  $\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2} = \frac{(12-1) \cdot 15^2}{10^2}$

$\chi^2 = 24.75$

Area = P-value =  $\chi^2_{cdf}(24.75, 99, 11) = .010$

CTS  $\chi^2 = 24.75$

By testing chart

P-value Method  $P\text{-value} \leq \alpha \Rightarrow H_0$  invalid

$.010 \leq .05 \Rightarrow H_1$  valid

Valid claim

Fail-to-reject the claim

May 16-7:25 AM

Given:  $n=10$ ,  $S=8$ ,  $H_0: \sigma \geq 8.5$ , claim is  $H_0$

$\alpha = .1$

Test the claim.

$H_0: \sigma \geq 8.5$  claim

$H_1: \sigma < 8.5$  LTT

CTS  $\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2} = \frac{(10-1) \cdot 8^2}{8.5^2}$

$\chi^2 = 7.972$

$df = n-1 = 9$

Area = P-value =  $\chi^2_{cdf}(0, 7.972, 9) = .463$

P-value  $> \alpha \Rightarrow H_0$  valid  $\Rightarrow$  Valid claim

$.463 > .1 \Rightarrow H_1$  invalid

FTR the claim

May 16-7:37 AM

Given:  $H_1: \sigma \neq 5$ , claim is  $H_0$ ,  $n=15$ ,  $S=7.5$   
 $\alpha = .02$

Test the claim.

$H_0: \sigma = 5$  claim

$H_1: \sigma \neq 5$  TTT

CTS  $\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2} = \frac{(15-1) \cdot 7.5^2}{5^2} = 31.5$   $df = n-1 = 14$

P-value

Find the area on each side,  
P-value = 2 \* Smaller area

$\chi^2_{df}(\alpha, df) = \chi^2_{df}(0, 31.5, 14) = .995$   $\chi^2_{df}(31.5, .99, 14) = .005$

P-value = 2 \* Smaller area  
 $= 2(.005) = .01$

P-value  $\leq \alpha$   $\Rightarrow H_0$  invalid  $H_1$  valid  
 $.01 \leq .02$

Invalid claim

**Reject the claim**

May 16-7:47 AM

The college claims that standard deviation of ages of all students is below 8 yrs.

$H_1: \sigma < 8$  LTT claim

I took a sample of 10 students, and standard deviation of their ages was 7.5 yrs.  
 $n=10$ ,  $S=7.5$

use  $\alpha = .1$  to test the claim.

$H_0: \sigma \geq 8$

$H_1: \sigma < 8$  LTT, claim

CTS  $\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2} = \frac{(10-1) \cdot 7.5^2}{8^2} = 7.910$

P-value

Area  $\Rightarrow$  Invalid claim

$\chi^2_{df}(\alpha, df) = \chi^2_{df}(0, 7.910, 9) = .457$

P-value  $> \alpha$   
 $.457 > .1$

$H_0$  valid  
 $H_1$  invalid

**Reject the claim**

May 16-7:58 AM

The college claims that standard deviation of all scores on final exams is not 10.  
 $H_1: \sigma \neq 10$  claim

I took a sample of 8 final exams, and standard deviation of their scores was 9.  
 $n=8, S=9$   
 $df=n-1=7$

Use this sample to test the claim. No  $\alpha \rightarrow$  use .05

$H_0: \sigma = 10$  CTS  
 $H_1: \sigma \neq 10$  claim TTT

$$\chi^2 = \frac{(n-1) \cdot S^2}{\sigma^2} = \frac{(8-1) \cdot 9^2}{10^2} = 5.67$$

P-value

Area =  $\chi^2_{df}(5.67, 7) = .579$   
P-value = 2 \* Smaller area = 2 \* .421 = .842

Area =  $\chi^2_{df}(0, 5.67, 7) = .421$   
P-value  $>$   $\alpha$   
.842  $>$  .05

$H_0$  valid  
 $H_1$  invalid claim invalid

**Reject the claim**

May 16-8:10 AM